

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
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1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE January 10, 1996	3. REPORT TYPE AND DATES COVERED Final Technical 1 Nov 92 - 31 OCT 95		
4. TITLE AND SUBTITLE  Theoretical Aerodynamics		5. FUNDING NUMBERS  F49620-93-1-022		
6. AUTHOR(S)  Julian D. Cole				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Office of Scientific Research 110 Duncan Avenue Suite B115 Bolling AFB, DC 20332-0001		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM 110 Duncan Avenue Suite B115 Bolling AFB DC 20332-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES				
11a. DISTRIBUTION/AVAILABILITY STATEMENT  <div style="border: 1px solid black; padding: 5px; text-align: center;"> <b>DISTRIBUTION STATEMENT A</b>  Approved for public release  Distribution Unlimited </div>		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words)  Mathematical and computational studies have been carried out on problems of theoretical aerodynamics. Shock free bodies and optimum critical airfoils have been considered in transonic theory, optimum three-dimensional lifting wings in hypersonic theory. Stability and transition of boundary layers has been analyzed according to triple deck theory. The Benjamin-Davis-Acrivos equation has been derived and it has been shown how solitons can lead to chaotic motion.  <div style="font-size: 2em; font-weight: bold; text-align: center;">19960122 031</div>				
14. SUBJECT TERMS  aerodynamics, stability and transition		15. NUMBER OF PAGES		
		16. PRICE CODE		
17. SECURITY CLASSIFICATION OF REPORT unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT unclassified	20. LIMITATION OF ABSTRACT	

# **Final Technical Report**

**January 11, 1996**

**Theoretical Aerodynamics**

**AFOSR Grant AF49620-93-1-0022DEF**

## **Introduction**

The objective of the research of the past three years has been to study and analyze a variety of mathematical problems arising in aerodynamic theory. The two principal areas under study were classical aerodynamics and stability and transition in boundary layers. Classical aerodynamics encompasses mainly transonic, second-order supersonic, and hypersonic theory. Stability and transition in boundary layers were studied on the basis of the asymptotic triple-deck theory. In general our methods combine numerical and asymptotic approaches. Our general aim is two-fold: first, to formulate new and significant problems for aerodynamic theory and second to generate new mathematical and computational ideas useful for solving these problems. The main results are documented in the publications (see below).

## 1. Transonic Aerodynamics

Studies leading to the design of optimum non-lifting and lifting critical airfoils have been completed. These airfoils have for a given constraint, such as thickness, the maximum critical (free-stream) Mach number. They are characterized by an arc on the surface on which the flow is sonic but there is no supersonic region or shock wave. These airfoils have better performance in this sense than conventional airfoils. This work is continuing in the study of cascades of such airfoils. Theory and calculations for this problem were carried out in the hodograph plane.

Shock-free bodies of revolution which are not symmetric fore and aft were designed. These are analogous to supercritical airfoils. The analysis and computation were carried out in a special hodograph plane for transonic small-disturbance theory (TSDT). These body shapes have a special utility because of the transonic area rule which states that for any configuration the drag (zero at design) is the same as that of the equivalent (same distribution of cross-section area) body of revolution.

Some simple examples for unsteady (TSDT) were studied in order to gain insight into the behavior of solutions to these equations. In particular, the problem of the flow due to the sudden deflection of a thin wedge was analyzed and it was shown how a non-uniformity of linearized theory leads to a transonic region in space-time. A similarity solution was formulated and a computational method was implemented to obtain the solution for the exactly sonic case. Subsonic and supersonic cases are natural extensions. The problem is relevant to the glancing incidence of a shock or Mach reflection.

The shock wave of a sonic boom was studied as it passed through a turbulent field. It was shown that transonic theory is necessary to describe the changes in the rather weak shock due to turbulence. The correct generalized TSDT equation was derived. Numerical calculations were carried out which showed substantial changes in the sonic boom compared to that in a non-turbulent atmosphere.

## 2. Hypersonic

The central problem considered was the design of optimum 3D hypersonic wings, according to hypersonic small deflection theory (HSDT). The analogy of 3D HSDT with the 2D unsteady Euler equation was used to set up a computer code. A series of problems was (and will be) considered leading to optimizing lift to drag ratio. Variations in conical twist were studied.

## 3. Stability and Transition; Separation

The utility of the triple-deck approach to the study of stability of boundary layers in various circumstances, subsonic, transonic, supersonic has been demonstrated. This asymptotic theory for large Reynolds number shows all essential features of the instability and transition process in both linear and nonlinear regimes. A systematic derivation of the Benjamin-Davis-Acrivos equation was given. The existence of solitons for this equation was documented and it was shown how a chaotic motion arises from this beginning.

Vortex-roughness interaction in the boundary-layer was studied and this showed great differences between wave packets and simple harmonic wave trains. Wave packets were shown to amplify extremely rapidly in contrast to wave trains.

The Landau-Goldstein singularity at a separation point of a boundary layer was analyzed. New possibilities for the behavior, compared to earlier results, were presented.

## Publications

1. Julian D. Cole, Donald Schwendeman, M.C.A. Kropinski, "Hodograph Design of Lifting Airfoils with High Critical Mach Numbers," *J. Theoretical and Computational Fluid Dynamics* **7**, 1995, pp. 173-188.
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3. Rusak, Z., Giddings, T.E. and Cole, J.D. "Interaction of a Weak Shock with Free Stream Disturbances," *AIAA Journal*, Vol. 33, 6, 1995, pp. 977-984.
4. O.S. Ryzhov & S. V. Timofeev, "Interaction of a potential vortex with a local roughness on the smooth surface," *J. Fluid Mech.* **287**, 1995, pp. 21-58.
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7. Schwendeman, D., Kropinski, M.C., and Cole, J.D., "An analytical study of optimal critical airfoils," *ICIAM Congress Proceedings*, Hamburg, 1995.
8. Rusak, Z. "Novel Similarity Solutions of the Sonic Small Disturbance Equation with Applications to Airfoil Transonic Aerodynamics," *SIAM Journal of Applied Mathematics*, **54** (2), 1994, pp. 285-308.
9. Rusak, Z. "Subsonic Flow Around the Leading Edge of a Thin Airfoil with a Parabolic Nose," *European Journal of Applied Mathematics*, **5**, 1994, pp. 283-311.

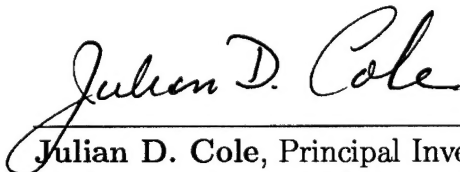
10. O.S. Ryzhov, "The development of nonlinear oscillations in a boundary layer and the onset of random disturbances," *Proc. IUTAM Symp. Nonlinear Instability of Nonparallel Flows*, Springer. To appear.
11. E.V. Bogdanova-Ryzhova & O.S. Ryzhov, "On singular solutions of the incompressible boundary-layer equation including a point of vanishing skin friction," *Acta Mechanica*, Vol. [Suppl]4, 1994, pp. 27-37.
12. E.V. Bogdanova-Ryzhova & O.S. Ryzhov, "On the nature of singularities inherent, under a given analytic distribution of the external pressure, in solutions of the Prandtl equations near the point of separation," *J. Theor. Comput. Fluid Dyn.*, 6, 1994, pp. 193-212.
13. Rusak, Z. "Subsonic and Transonic Flow Around the Leading Edge of a Thin Airfoil with a Parabolic Nose," *SIAM Series: Frontiers in Applied Mathematics: Transonic Aerodynamics - Problems in Asymptotic Theory*, edited by Pamela L. Cook, SIAM Publication, March 1993, pp. 8-28.

## Theses

1. M.C.A. Kropinski, "A Study of Optimal Critical Airfoils," Ph.D. Thesis, Rensselaer Polytechnic Institute, June 1993.
2. R. Buckmire, "Design of Shock Free Transonic Slender Bodies of Revolution," Ph.D. Thesis, Rensselaer Polytechnic Institute, December 1993.

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